Consensus-Based Image Segmentation via Topological Persistence

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Motivation

- It is difficult for a single algorithm with the same parameters to segment all the images successfully due to variations between images.
- The desired boundaries are detected more consistently than other boundaries.

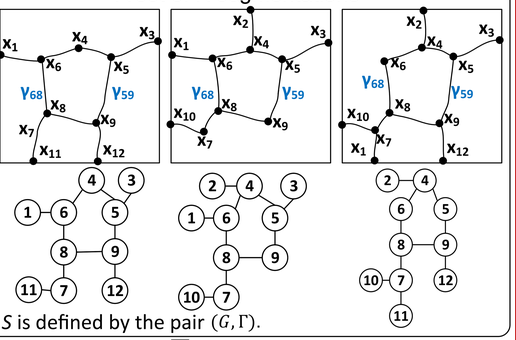
Methodology

Segmentation Model

A set of K segmentations

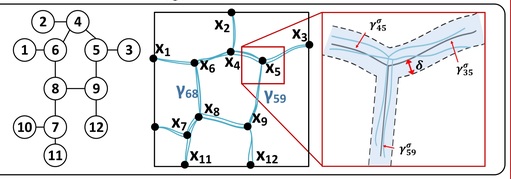
A segmentation is presented by a set X of nodes x_i and a set Γ of curves γ_{ij}.

Corresponding graph $G = (V, E)$.

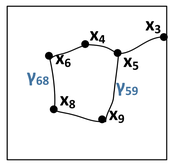


A segmentation S is defined by the pair (G, Γ).

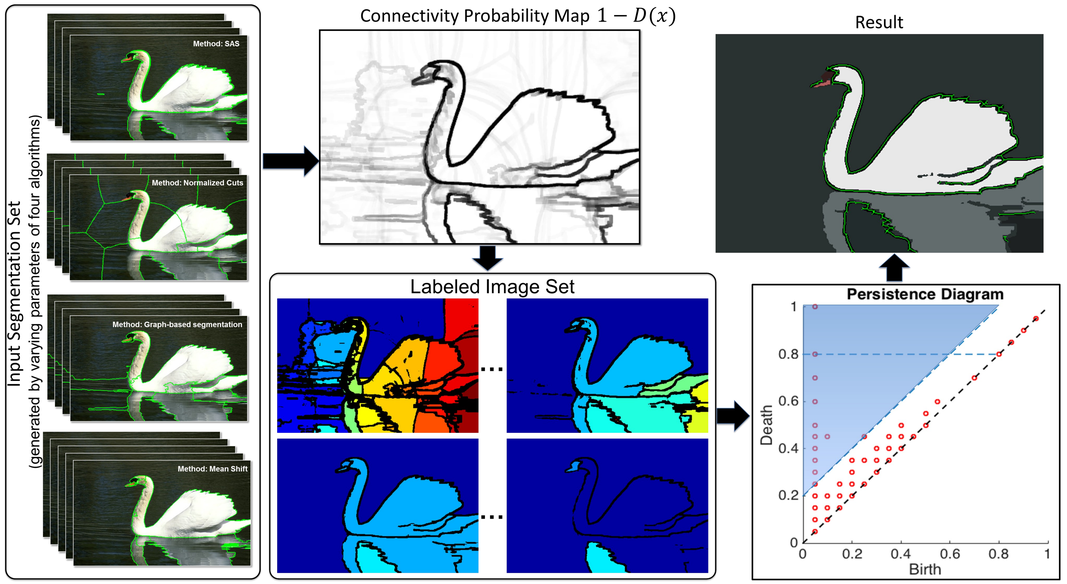
Over-segmentation



- p_{ij} is defined as the probability of the curve γ_{ij} being present in a segmentation.
- Assume for all the edges with $p_{ij} > \sigma$, there exists a constant δ and a set Γ_σ of curves γ_{ij}^σ that $|\gamma_{ij} - \gamma_{ij}^\sigma|_\infty \le \delta$.
- Our objective is to estimate $S_\sigma = (G_\sigma, \Gamma_\sigma)$.

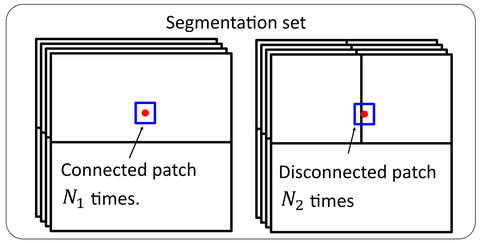


Theorem. *The set* $C = \{x | D_\delta(x) \ge \sigma\}$ *satisfies:* $im(S_\sigma) \subset C$ *and* $im(S_\sigma)^c \ominus B_{2\delta} \subset C^c$, *where* $im(S_\sigma) = \cup_{(i,j)\in E_\sigma} im(\gamma_{ij}^\sigma)$, $im(\gamma_{ij}^\sigma) = \{x = \gamma_{ij}^\sigma(s) \in \Omega | s \in [0,1]\}$, $D_\delta(x) := P[im(S) \cap B_\delta(x) \ne \emptyset]$, $B_{2\delta}$ *is the ball of radius* 2δ *and* $\ominus$ *is the morphological erosion operator.*



Boundary Characterization

$D(x)$ is estimated by an $n \times n$ patch centering at x.



$$D^*(x) = \frac{N_2}{N_1 + N_2}$$

Connectivity probability map is obtained by estimating $D(x)$ for all x in the image.

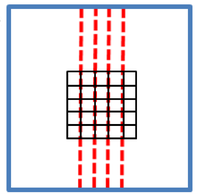
Choice of Parameter n

Let 2σ be the range of perturbation of an edge.

Case 1

$n - 2 \ge 2\delta, D_n^*(x) = p_{ij}$.

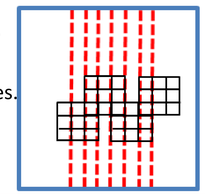
The range can be covered by a single patch.



Case 2

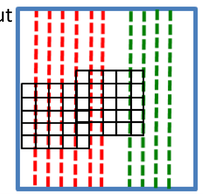
$n - 1 < 2\delta$, $p_{ij}/m \le D_n^*(x) < p_{ij}$, where $m = [(2\delta + 1)/(n - 1)]$.

The range can be covered by m patches.



Case 3

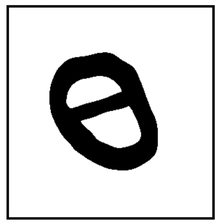
Perturbation range of two edges n-1 pixels away from each other can be identified without the influence of other edges.



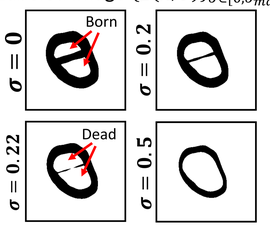
Topological Persistence

- **Extract robust regions by using topological persistence**

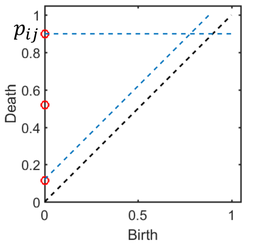
Probability map



Labeled image $\{L(n,\sigma)\}_{\sigma\in[0,\sigma_{max}]}$



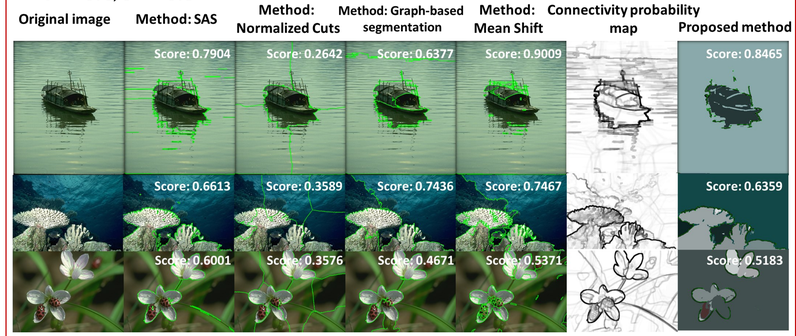
Persistence Diagram



- Apply a persistence threshold τ to avoid the connected components generated by noise. Set to maker $\sigma_{max} = p_{ij}$ sure capture the curves present with probability greater than p_{ij} within a range $2\delta \le n - 2$.

Results

- $\tau = 0.4, \sigma = 0.8$



Future Work

- Consider multiple neighborhoods of size n x n (treating n as another parameter).
- Find ways to select an appropriate range of parameters for the base segmentation algorithms.
- Obtain a good estimate for the persistence threshold via training using user-input.
- Identify better metric to qualify the quality of a segmentation.