Non-Rigid Image Registration under Non-Deterministic Deformation Bounds

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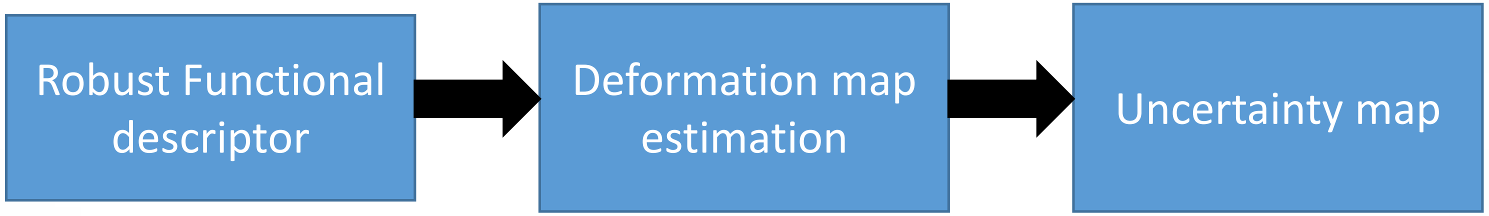
Motivation

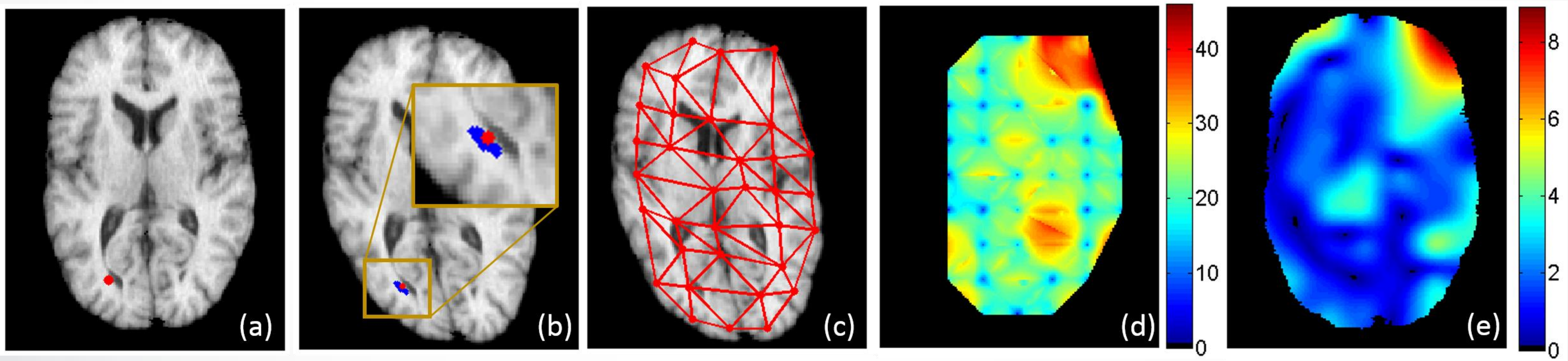
- Image Registration Challenges

➢ Most of deformation in real world is non-rigid

➢ Matching under rigid deformation is well studied, but it does not work for non-rigid deformation, since this kind of transformation is arbitrary.

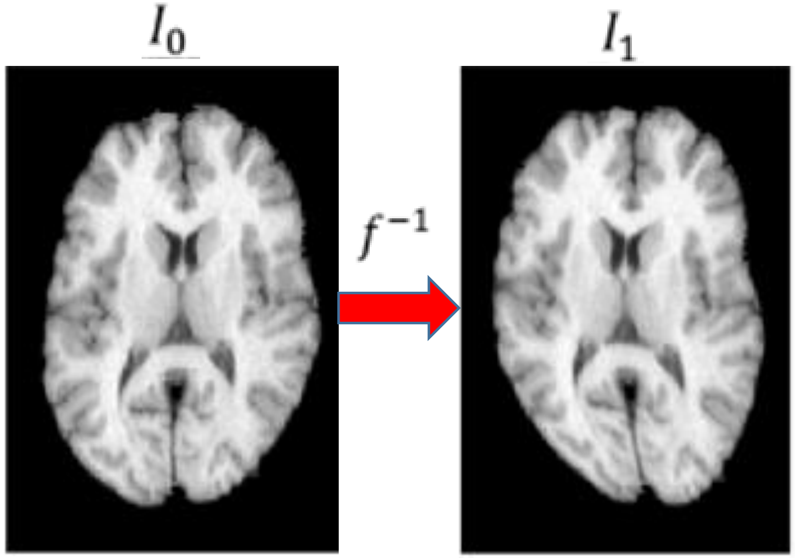
Pipeline





Registration Approach: (a) Test point x0 (in red) on image before deformation. (b) Corresponding feasible matching set Sx0 (in blue) and true matching (in red). (c) Delaunay triangulation of selected match points. (d) Uncertainty bound in pixels (image has size 149 x 188). (e) Difference in pixels between truth and estimated deformation maps.

Image Model



$$I_1 \circ f_{10}(x) = I_0(x)$$

where $f_{10} : \Omega_0 \to \Omega_1$ is a homeomorphism (with inverse f_{01}) that satisfies:

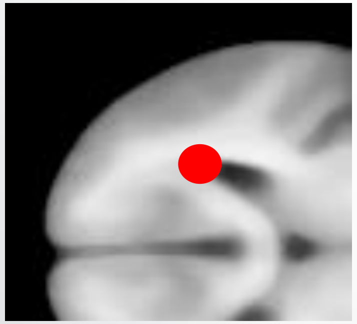
$$(1 - K_d)\|f_{10}(x) - f_{10}(y)\| \leq \|x - y\| \leq (1 + K_d)\|f_{10}(x) - f_{10}(y)\|$$

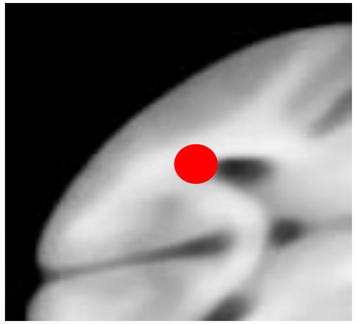
for all $x, y \in \Omega_0$ where K_d is a factor bounding the unknown deformation.

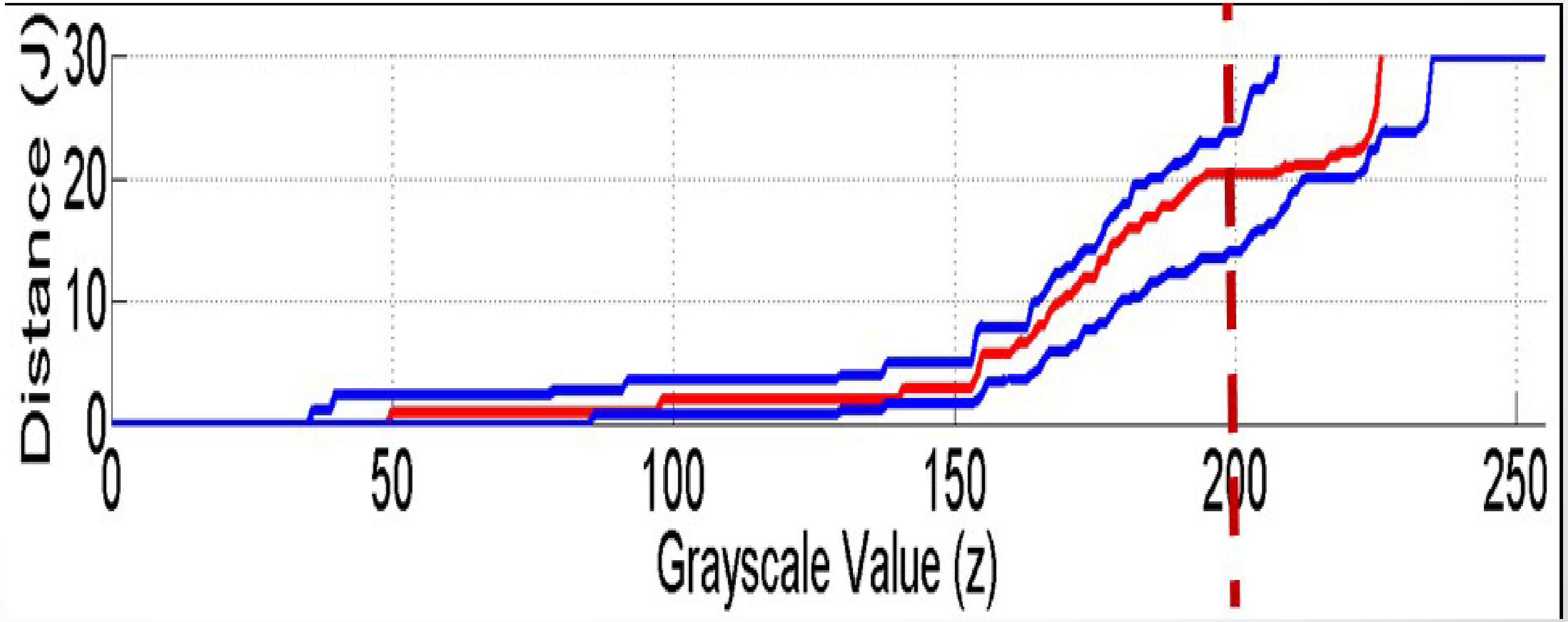
Functional Descriptor

Original image

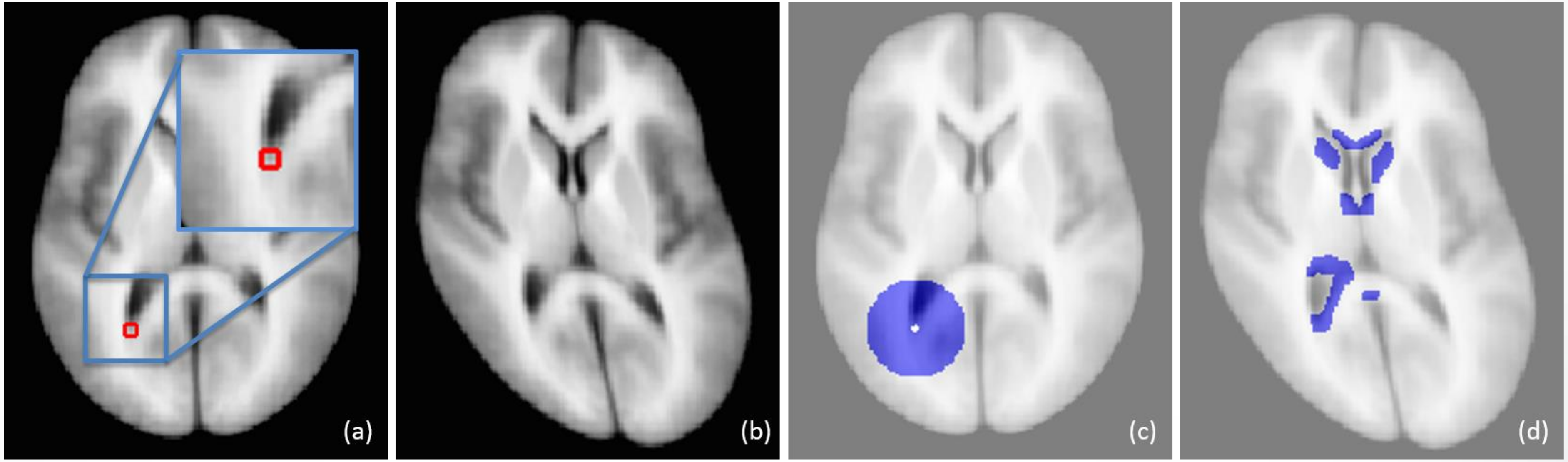
Deformed image







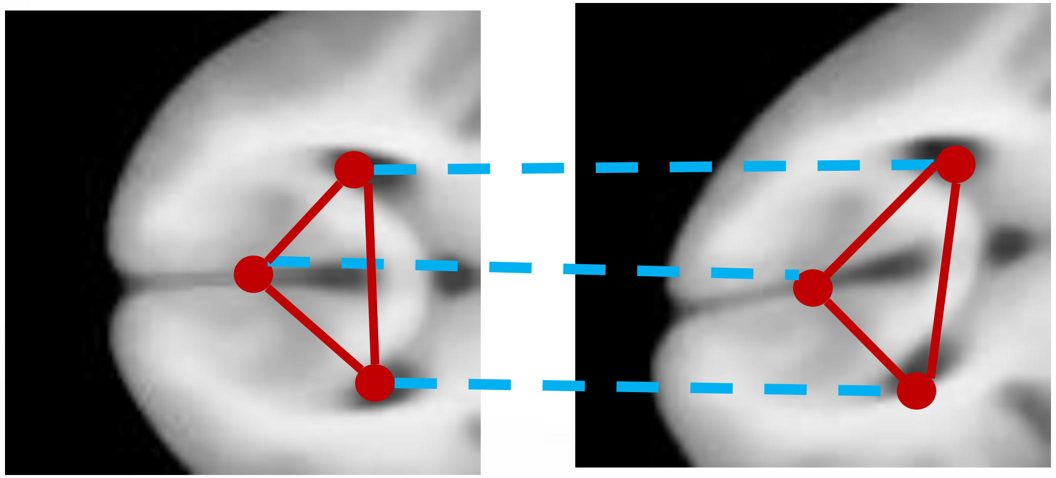
Single Point Matching Process (Kd = 0.2)



(a) Test point (in red square). (b) Deformation image with maximum 20% deformation. (c) Support region of the descriptor (blue circle with radius 20 pixels). (d) Potential matching set (in blue) for this test point.

Extend to Map Estimation

To achieve map estimation, we can construct a graph to present a region, then estimate the mapping by the corresponding points.



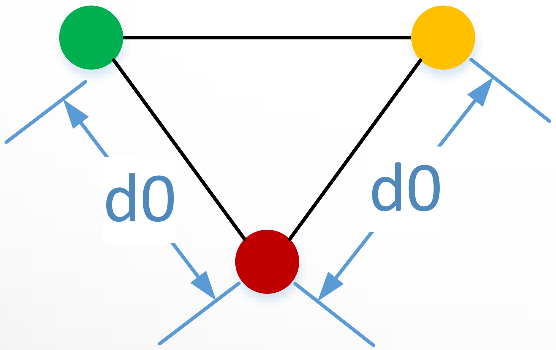
However, there maybe more than one potential matching regions for one test point. To remove as many as wrong potential matches, a geometry constraint is applied.

Geometry Constraint

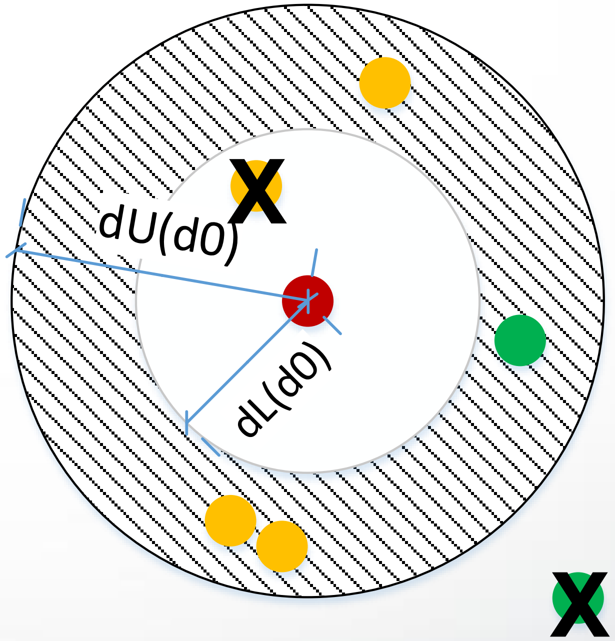
According to our image model, any two points after deformation should satisfy:

$$(1+K_d)^{-1}\|x_0-y_0\| \leq \|f_{10}(x_0)-f_{10}(y_0)\| \leq (1-K_d)^{-1}\|x_0-y_0\|$$

Any potential matches which do not satisfy this can be removed.

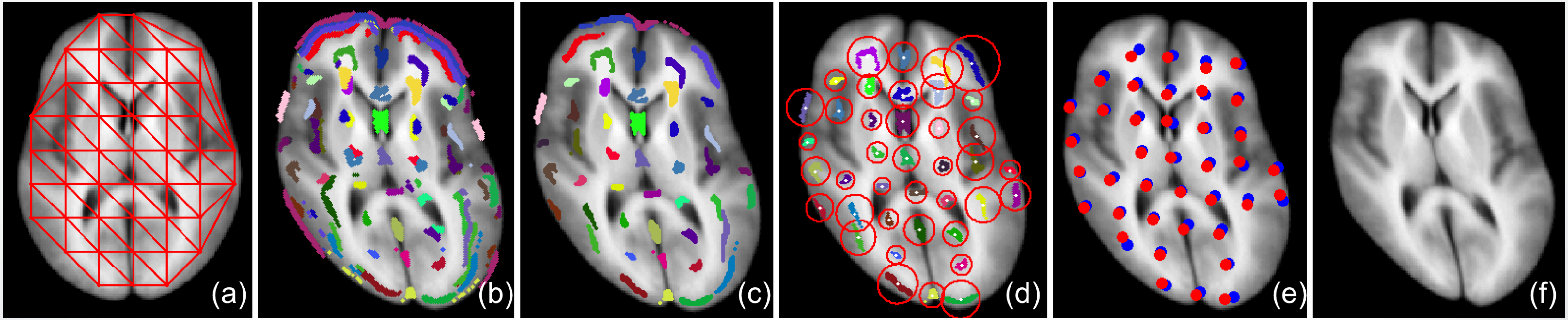


Original image



After deformation

Map Estimation



Map Estimation: (a) Test set P_0 and Delaunay triangulation. (b) Potential matching sets $S_{x0,i}$ for set P_0. Different colors denote different test points in P_0. (c) Potential matching sets $S'_{x0,i}$ for set P_0 after applying distance constraint. (d) Potential matching sets $S'_{x0,i}$ for set P'_0 and the minimum circle covering each $S'_{x0,i}$ with radius less than γ. (e) Estimated matching points (in red) and groundtruth (in blue) for set P'_0. (f) Estimated deformation image.

Uncertainty Computation

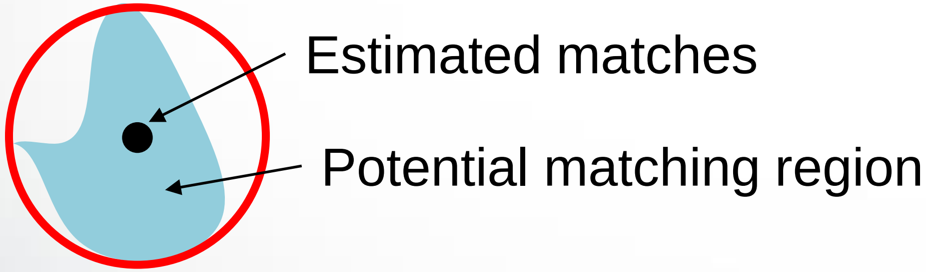
- It is often assumed that the registration images are properly aligned, which can introduce some bias on the statistical models.

- However, if a bound on uncertainty on the registration map was provided, this could be built into the modeling tools leading to more accurate statistical models

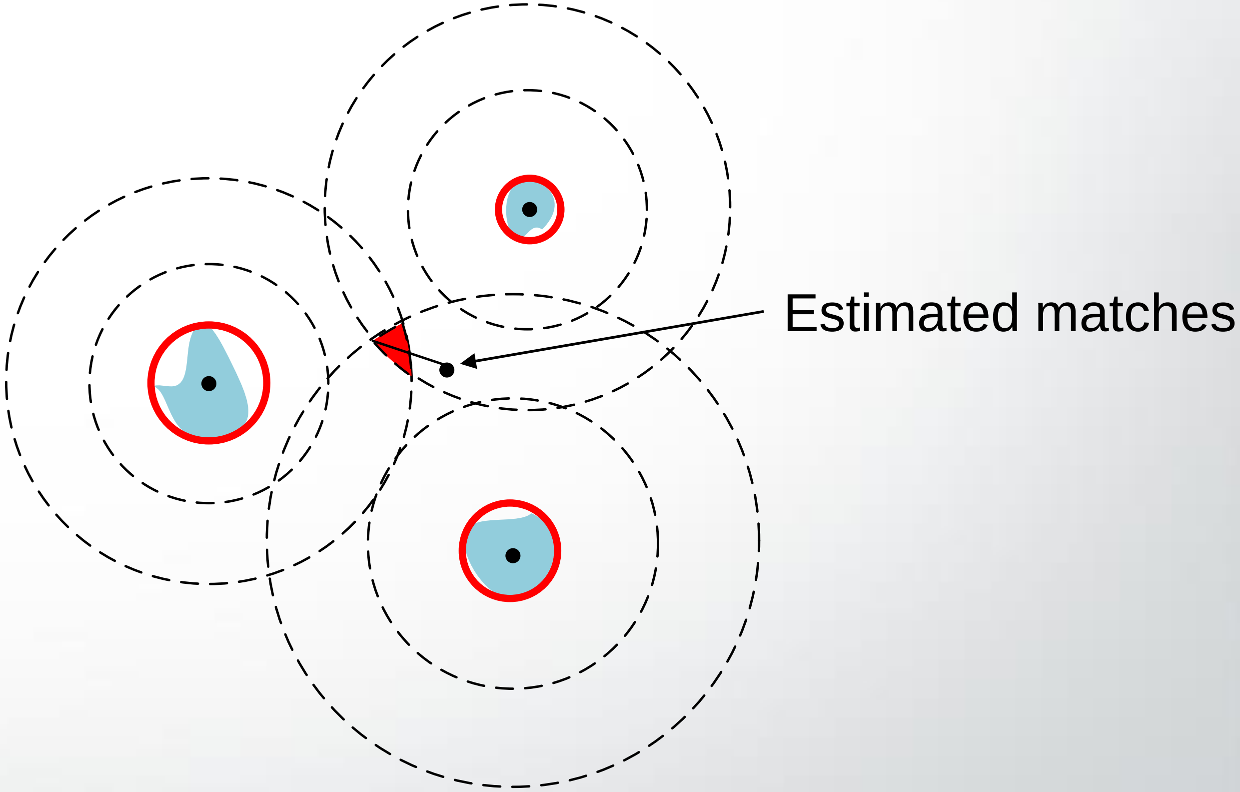
Uncertainty Computation

The definition of uncertainty is the largest possible distance between true matching and the estimated matching point.

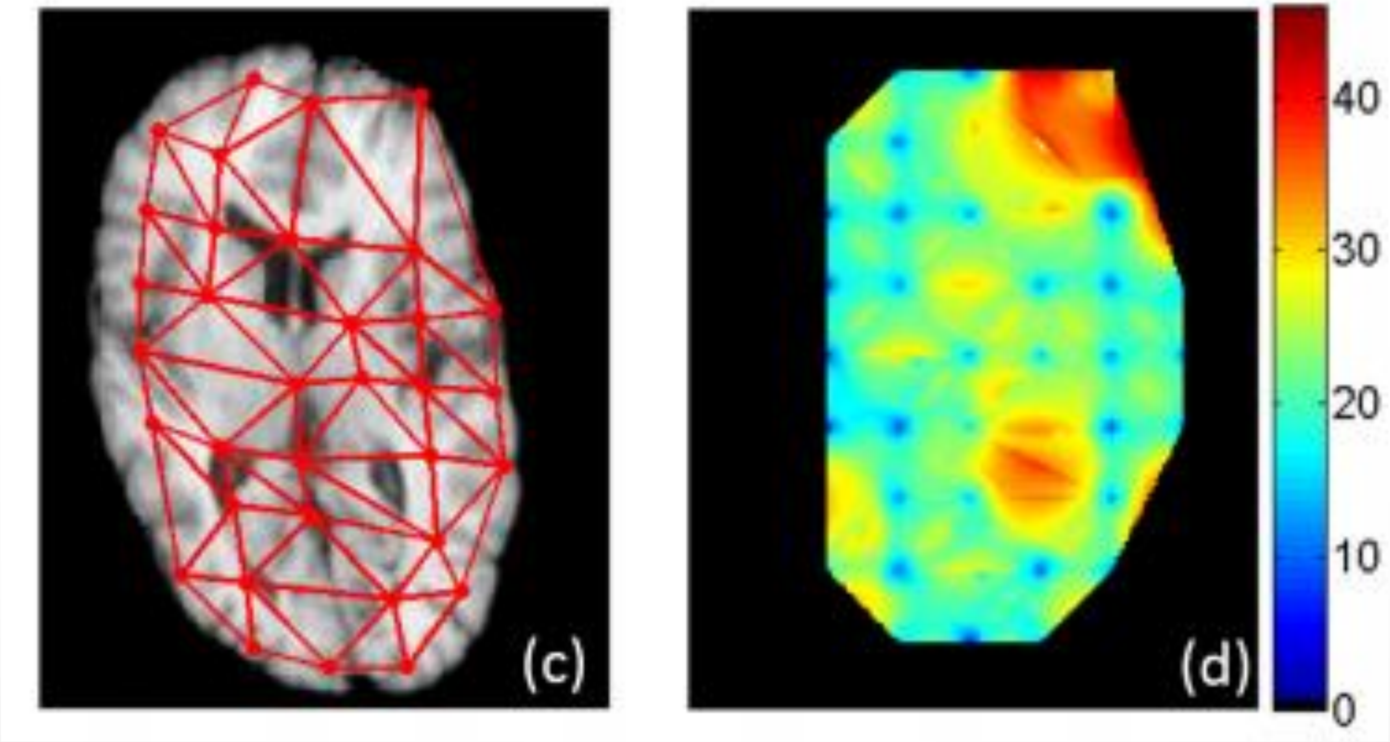
Test point



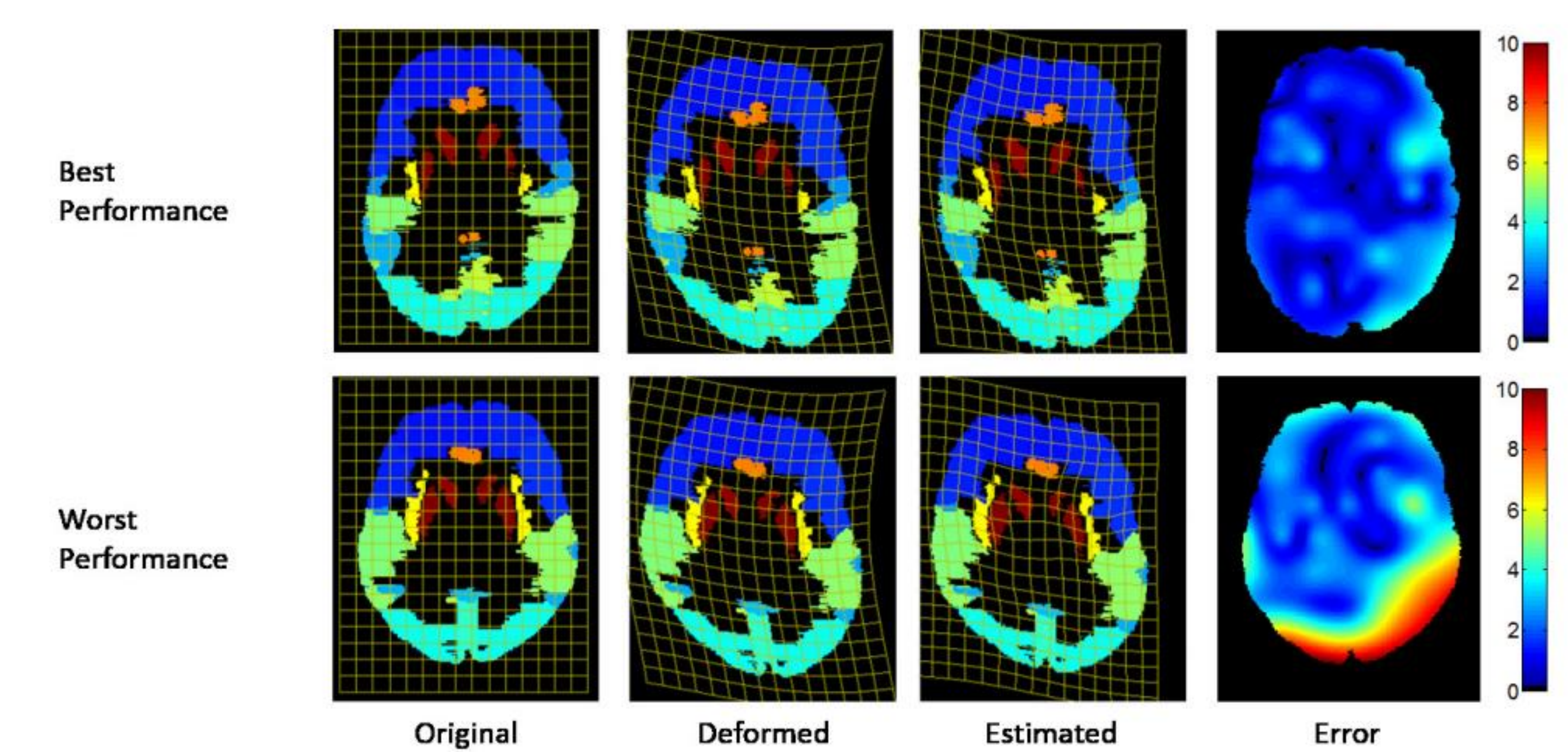
Remaining point



Uncertainty Computation



Result



Results of proposed method on the labeled LPBA40 dataset with 20% deformation.

Evaluation metrics

$$ME = \frac{1}{|\Omega_0|} \int_{\Omega_0} ||f_{10}(x) - \hat{f}_{10}(x)||_2 \, dx$$

Mapping Error

where $|\Omega_0|$ is the area of Ω_0, f_{10} is the true map and $\hat{f}_{10}$ is the estimated map.

$$TOS = \frac{\sum_r |\mathcal{R}_r \cap \hat{\mathcal{R}}_r|}{\sum_r |\mathcal{R}_r|}$$

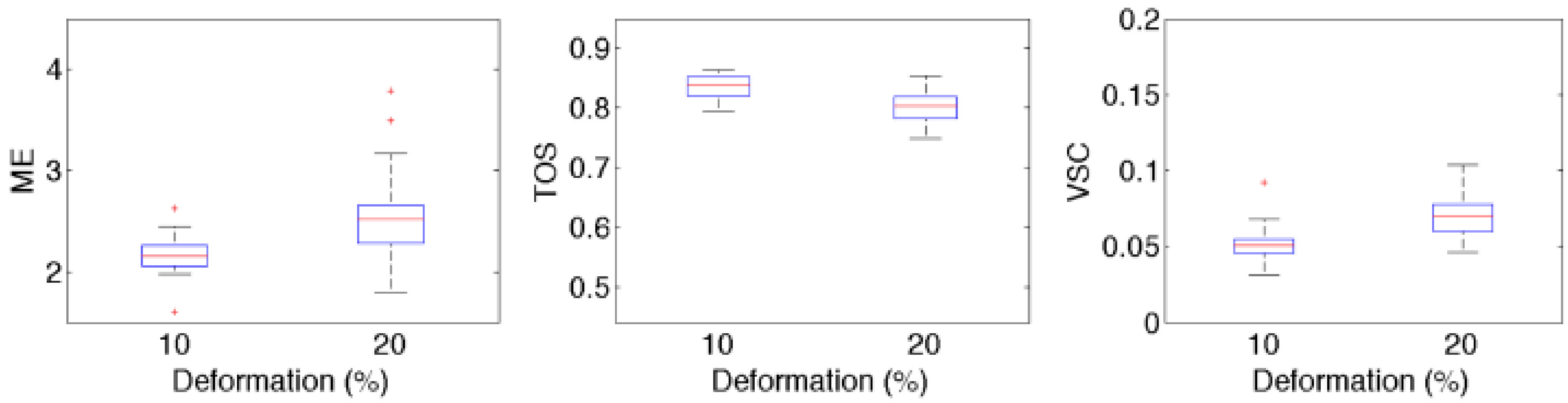
Target Overlap Score

$$VSC = 2\,\frac{\sum_r (|\mathcal{R}_r - \hat{\mathcal{R}}_r|)}{\sum_r (|\mathcal{R}_r| + |\hat{\mathcal{R}}_r|)},$$

Volume Similarity Coefficient

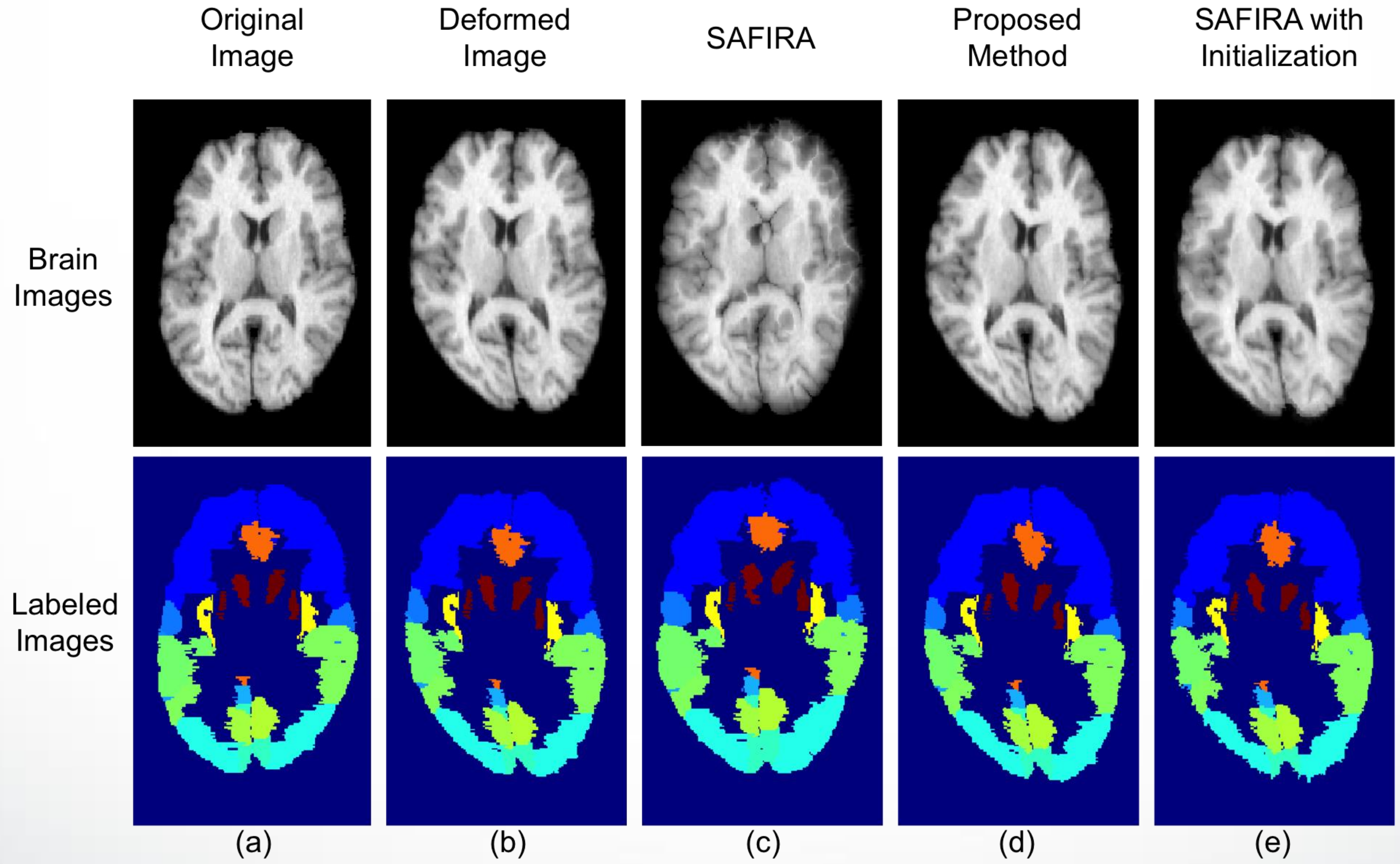
where $\mathcal{R}_r$ and $\hat{\mathcal{R}}_r$ denote the $r-$th true and estimated region in the brain respectively.

Evaluation of Proposed Method



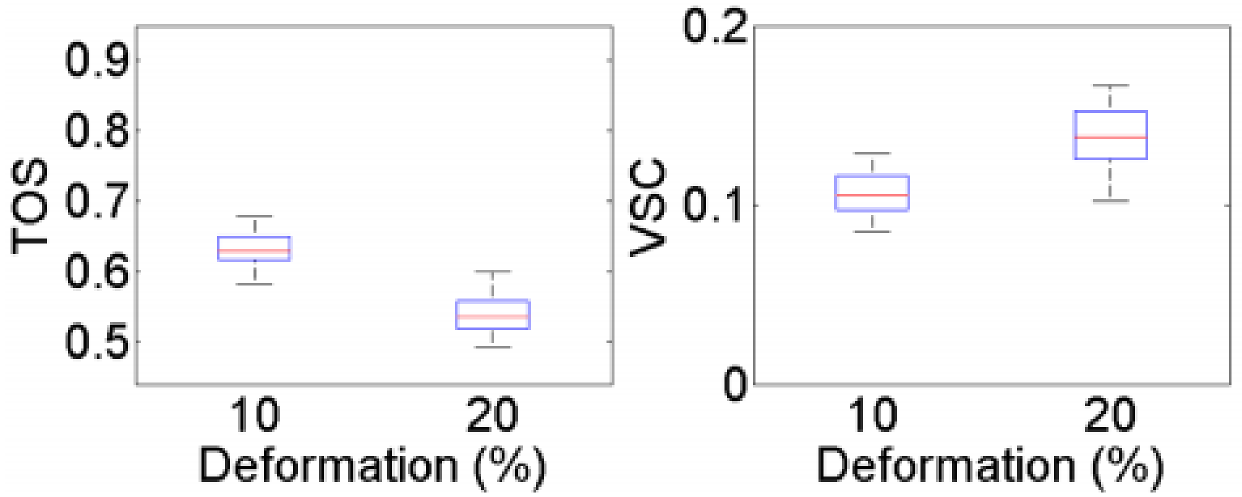
Mapping Error (left), Target Overlap Score (middle) and Volume Similarity Coefficient (right) metrics applied to the LPBA40 dataset with 10% and 20% deformation.

Our Method as Initialization of Registration

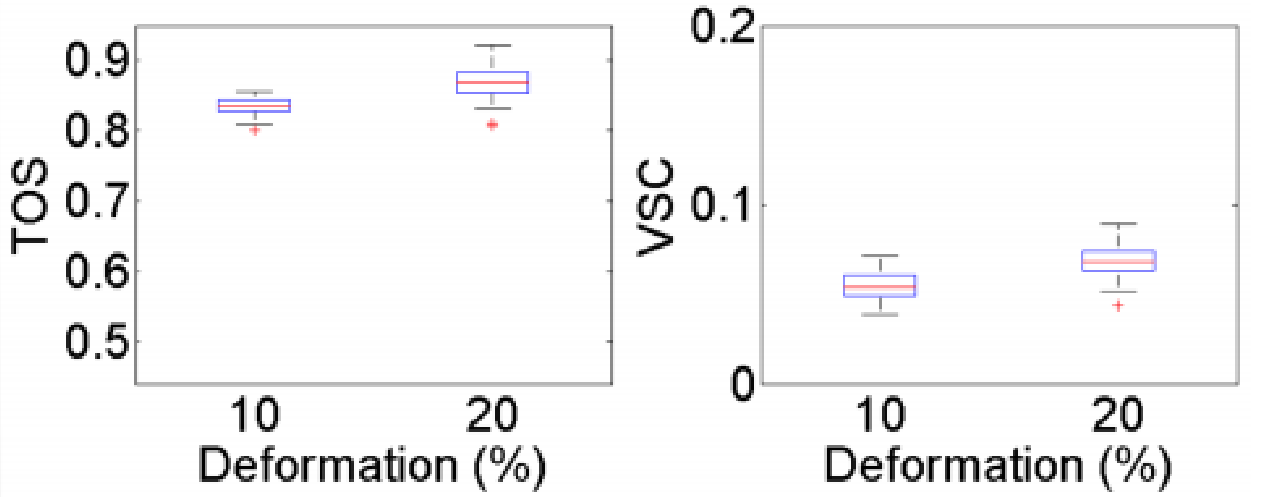


Proposed Method as Initialization of Registration

SAFIRA



Our method as initialization to SAFIRA



Conclusion

- A new non-rigid registration methodology is introduced which makes use of a Lipschitz deformation model in order to provide guarantees on the registration results.

- The uncertainty on the estimated deformation is derived from the size of feasible matching sets that are computed for individual point matches.

- In the future, we aim to extend our approach to provide not only a single estimated map with large uncertainty, but also a few candidate maps with lower uncertainty, in order to provide more refined outcomes.

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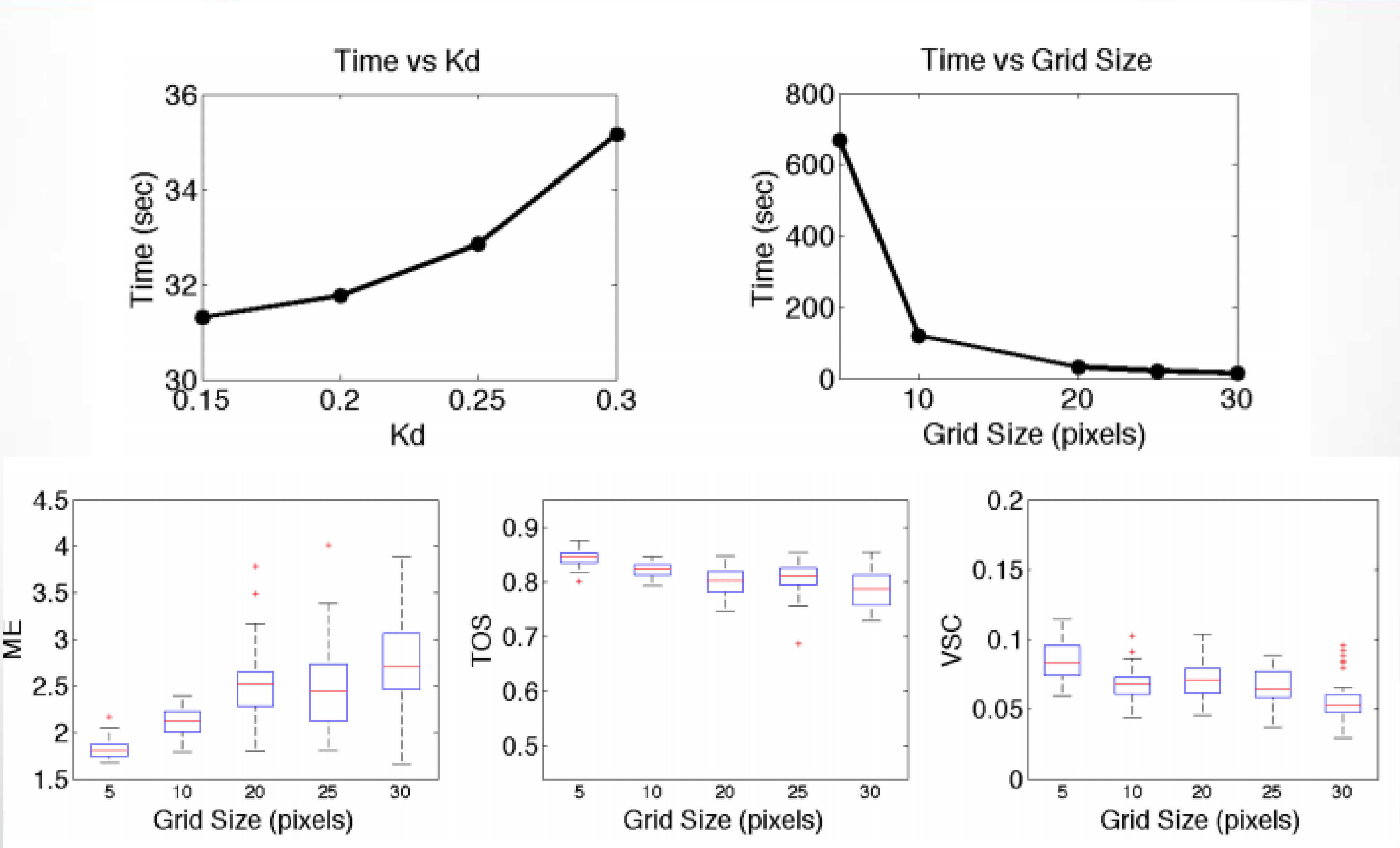
Acknowledgement

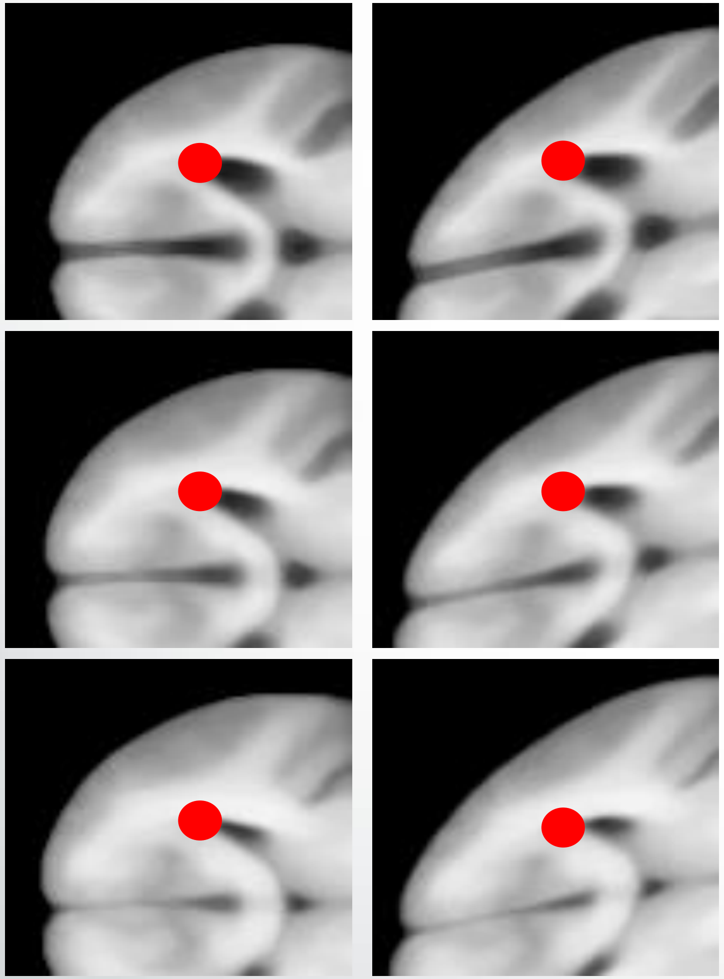
We would like to thank Dr. Natasha Lepore from University of Southern California and Dr. Helene Nadeau from Dawson College for providing us the SAFIRA code and information to run the code for comparison.

Thank you

$$f_{01}\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} z_1 + 0.5c\ \cos(0.02 z_2) \\ z_2 + 0.5c\ \cos(0.02 z_1) \end{pmatrix}$$


Time vs Kd
Time (sec)
Kd
Time vs Grid Size
Time (sec)
Grid Size (pixels)
ME
Grid Size (pixels)
TOS
Grid Size (pixels)
VSC
Grid Size (pixels)

Original image Deformed image



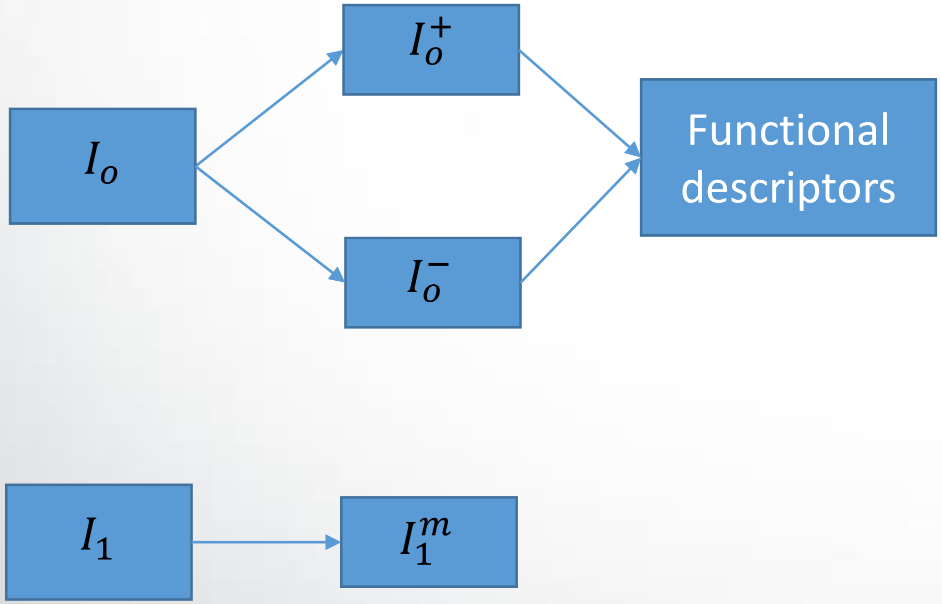
$$F_{k,B(x,r)}(s) = \frac{1}{\pi r^2} \int_{B(x,r)} \mathbb{1}_{I_k^{-1}(-\infty,s]}(y)\,dy \quad \text{for} \quad k \in \{0,1\}$$

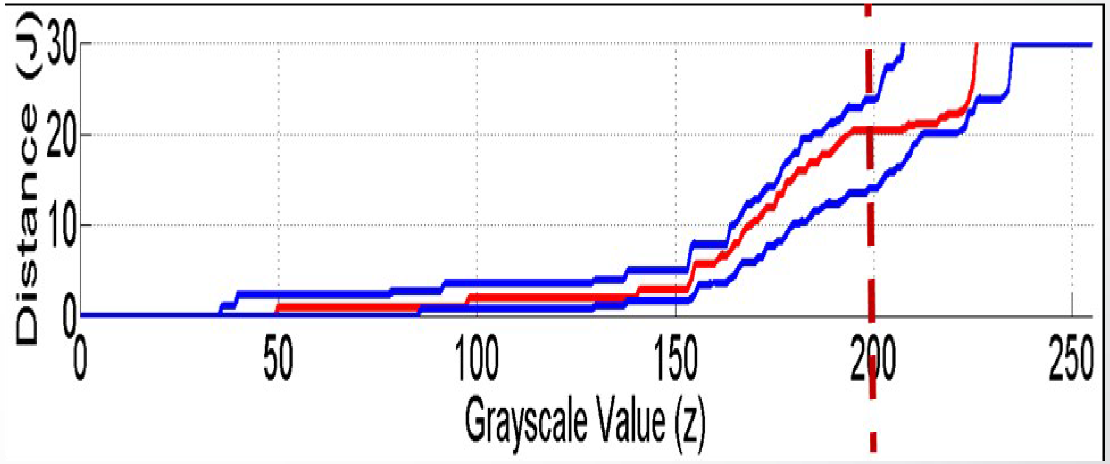
$$J_{0-}(x) = F^{-1}_{0,B(x,(1+K_d)\rho)}(\tau_-), \quad J_0(x) = F^{-1}_{0,B(x,\rho)}(\tau), \quad J_{0+}(x) = F^{-1}_{0,B(x,(1+K_d)\rho)}(\tau_+),$$

$$J_{1-}(x) = F^{-1}_{1,B(x,(1-K_d)^{-1}\rho)}(\tau_-), \quad J_1(x) = F^{-1}_{1,B(x,\rho)}(\tau), \quad J_{1+}(x) = F^{-1}_{1,B(x,(1-K_d)^{-1}\rho)}(\tau_+)$$

Robust pre-processing and functional descriptor

- The idea is to pre-process in a way that guarantees the descriptors will be robust to unknown deformation, noise and discretization process.
- We perform three different types of process: one is median, the other two are similar to dilation and erosion.
- In the plot, blue curves come from "dilation" and "erosion" images, and red curve comes from median filtered image.





Proposed Method as Initialization of Registration

Method	10%		20%	
	TOS	VSC	TOS	VSC
SAFIRA (40 iterations)	0.6313	0.1060	0.5396	0.1383
Proposed Method	0.8351	0.0516	0.7999	0.0701
SAFIRA with Initialization (40 iterations)	0.8340	0.0545	0.8671	0.0695

TOS and VSC scores for three different registration approaches applied to LBPA40 dataset